2

Exemplary Practice: What Does It Look Like?

While I was helping my daughter Helen with her math homework, I asked her to explain why she chose the operation she used to solve a problem. She not only did not know, but also did not care. She was more interested in getting the right answer by plugging in the proper formula. This is how she was taught math, and she doesn't seem to want to change the way she learned it. I can only generalize that this is how many students are responding to attempts by teachers to create conceptual understanding. This age group is where so many students lose interest in math—just when they should be finding the beauty of it. Perhaps I shouldn't worry too much; Helen's passion lies in social studies and literature. She is not a "math-brained" child, I guess. Are these children born and not made that way?

Angeline
Preservice teacher

Like Helen, many students today perceive mathematics to be a bunch of numbers that plug into formulas to solve problems. More often than not, the problems they are asked to solve are not *their* problems, nor do they come close to something they are interested in pursuing.

Helen has two problems, which are indicative of the problems facing too many students today. She not only does not like mathematics, but also has influential teachers who accept this as a natural outcome. What happens to Helen's mathematics learning when her mom or her teacher believes that she does not have a "math brain"? If they decide that it is acceptable for Helen not to succeed in mathematics because she is smart in other areas and that there is no reason to work to enhance her mathematical understanding, then Helen may never change her own attitude about mathematics. On the other hand, if the belief that

Helen is not "math-brained" encourages her mother and her teacher to reason that because she is strong in some areas, they should work toward connecting the mathematics that she is learning to her areas of interest in a way that makes sense to her, then Helen has a fighting chance to understand, appreciate, and perhaps even love mathematics. The reality is that every student has a unique and complex brain; our classrooms are composed of many Helens with many varying interests and aptitudes. The National Council of Teachers of Mathematics (NCTM) has made recommendations for addressing the diverse needs of students in today's classrooms.

THE PRINCIPLES AND STANDARDS FOR SCHOOL MATHEMATICS

The recommendations for reforming curriculum, teaching, and assessment made by the NCTM provide a vision of what a classroom influenced by reform principles should look like to reach Helen and other students. In 1989, NCTM recommended that we teach and assess students in very nontraditional ways. These goals are reiterated and others updated in *Principles and Standards for School Mathematics* (NCTM, 2000; hereafter called *Principles and Standards*), which depicts the vision and directions for school mathematics programs. The overall purpose of the *Principles and Standards* is to revise and clarify the unique trilogy of NCTM standards published in 1989, 1991, and 1995 (hereafter called the *Standards* documents) that defined standards for content, teaching, learning, and assessment of K–12 mathematics programs.

Past NCTM President Glenda Lappan highlighted the key components as follows: "What is the reform of mathematics teaching and learning guided by NCTM's Standards all about? My answer is that we are about the following three things: upgrading the curriculum, improving classroom instruction, and assessing students' progress to support the ongoing mathematics learning of each student" (Lappan 1998, 3). Lappan further noted that two commitments inform NCTM's reform efforts: inclusiveness and understanding. All students should experience effective mathematics and teaching, and the focus of mathematics instruction should be to help students develop a deep understanding of important mathematics concepts (3). Support of both commitments requires that teachers believe that all students, regardless of their personal characteristics, backgrounds, or physical challenges, can learn challenging mathematics. It also requires that teachers know how to probe current understandings of students and that they can present students with engaging tasks which may help them connect new knowledge to old knowledge. It is important to note that teachers are to apply the equity principle to all students: "Equity does not mean that every student should receive identical instruction; instead it demands that reasonable and appropriate accommodations be made as needed to promote access and attainment for all students" (12). The theory of constructivism is useful in efforts to implement these commitments because it is practiced or experienced in an environment in which learners are trying to make sense of a problematic situation in order to understand an idea. Constructivism is a framework for NCTM's reform efforts.

Principles and Standards consists of six principles and ten standards that describe characteristics of quality instructional programs and valued goals for students' mathematical knowledge. Together they form the basis for developing effective mathematics instruction within four grade bands: prekindergarten through grade 2, grades 3–5, grades 6–8, and grades 9–12.

Principles and Standards builds on the solid foundation provided in the NCTM *Standards* documents through a set of six principles that address the question: What are the characteristics of mathematics instructional programs that will provide all students with high-quality mathematics education experiences across the grades? Six characteristics, called *guiding principles*, are offered as basic tenets on which to establish quality programs and guide decisions about mathematics instruction at all levels: These focus on equity, curriculum, teaching, learning, assessment, and technology.

Ten standards address the question: What mathematical content and processes should students know and be able to do as they progress through school? Of the ten, five are mathematical content standards that describe what students should know and be able to do within the areas of numbers and operations, algebra, geometry, measurement, data analysis, probability, and statistics. The other five are process standards that address students' acquisition, growth in, and use of mathematical knowledge in the areas of problem solving, reasoning, connections, communication, and representation.

THE NEW BASIC SKILLS

It is important, however, that throughout the teaching of the standards, teachers reinforce students' mastery of the basic skills. In his first address as NCTM President, Lee Stiff (2000) strongly supported this statement:

> NCTM has always argued for a strong foundation on learning the basics. Our vision of basics, however, goes beyond mere number-crunching skills. We hope *Principles and Standards* will help educators, school boards, parents, and business leaders recognize that the new economy demands greater and more sophisticated mathematical knowledge. "Shopkeeper's math" alone is not enough in a high tech environment. NCTM's vision of school mathematics prepares students to meet the challenges that lie ahead in a future they can't imagine" (3).

Hereafter, we refer to reformed teaching of the basics that is taught from the perspectives of the *Principles and Standards* as the "new basic skills."

ENVISIONING REFORM-BASED CLASSROOM ENVIRONMENTS

Not surprisingly, creating a coherent curriculum and classroom environment to promote such reforms is not easy, partly because acquiring a clear vision of the key elements and how they interrelate requires new ways of thinking, as well as practice, guidance, and time to evolve. Teachers or curriculum writers must therefore exercise caution against a limited vision of the *Principles and Standards* that might lead to a superficial or misguided application. For example, look at the following lesson in a seventh–ninth grade class and ask, "How different are the teaching, instructional activities, and student participation from those of a traditional classroom?"

> The bell rings and Nancy's students enter class. They quickly sit in their assigned groups of four and take out their calculators. Nancy's goal for the class is to have them model addition of integers with colored counters. She begins with a review of the properties of integers and their representations with the counters, and then gives each student a package of counters and a work sheet on addition of integers. Students decide who will tackle which problem, and the groups set to work. Nancy visits each group to monitor its progress.

This description includes many of the concepts that we associate with reform: The students are working in groups with manipulatives that include calculators, and the teacher monitors progress. How could the lesson not be reform-based? Let us take a closer look.

> In her discussion of the colored counters, Nancy first defines the use of the counters: a black counter represents a positive number, and a white is a negative number. Hence, three black counters represent positive three. Next she tells students how to add integers having the same signs and then models the example with the counters: "To add two integers with the same sign, just add the numbers and keep the sign. So, (+2) plus (+3) equals (+5), and we can show this is true with the counters." She draws three black counters and adds two more blacks to show a total of five black counters or +5. She next explains how to add when signs are different: "If the signs are different, then subtract the two numbers and take the sign of the number with the larger absolute value. So what do we get for (–3) + (+4)?" A student gives the correct answer of +1, and Nancy then draws three white counters and four black counters on the board to verify the answer. A student asks, "Why do we have to use the counters if we can get the answer by using your rules first anyway?" Nancy responds that this is just another way to do such problems. As she hands each student a sheet with exercises on addition of integers, Nancy instructs them to use the counters to show

> the results of their actions. Students decide who will do which problem and begin working. Some use calculators with their worksheet. When most are finished, they wait for other students to finish working. Nancy visits each group, correcting any errors. She assigns different students to put problems on the board when the groups finish.

Our closer scrutiny shows that what looks like reformed teaching lacks key ingredients of reform. First, consider Nancy's use of manipulatives. Properly used, manipulatives provide an alternative, concrete representation that is conducive to students' initial understanding of more abstract concepts or algorithms. They are valuable when introduced as an integral part of a lesson to foster conceptual understanding by helping students see patterns. Nancy's use of the colored counters does neither because she presents them from an algorithmic perspective. Yet colored counters are helpful to students' discovery of the rules for operations on integers. To help students discover them, Nancy would have had to connect her introduction of the colored counters to that of integers as representations of opposite situations using an appropriate model. For example, using a win-lose model translates "+3 dollars" to mean, "I won three dollars," and "–2 dollars" to mean, "I lost two dollars." The end result from adding is "I won 3, then lost 2, so I am left with only one dollar," which we represent as +1.

The concept of opposite numbers follows easily since winning three dollars and then losing three dollars neutralize each other and yield zero: (+3) + (–3) = 0. Having explained this model to the class, Nancy could then use it to develop the rules, or she could, at this point, introduce colored counters as a visual approach to operations with signed numbers. She could say, "Let black counters represent positive numbers and white counters represent the negatives. How can we represent (+3), (–2), or 0? Consider this circle representing a set of colored counters where each black is matched with a white. What number does that represent? Let's define addition: to add two numbers is to combine counters representing the numbers inside the circle and then to eliminate zeros. Let's go back now and use counters to find (+3) + (–2)." Once students eliminate zeros, they should be encouraged to see that only one black counter, or +1, remains as the answer. Nancy could then have students practice adding single digit integers that they create at random.

After some practice, telling students to add (+234) + (–456) without a calculator should either get a student to propose a rule which other students should test, or it should create the need for students to find a rule. From this point, students are ready to proceed in an organized manner to seek patterns. Nancy could ask students for suggestions on how to proceed or have students complete a worksheet with a sequence of problems conducive to generating the rules. Once students have discussed and found some patterns, they should test them with several examples, and if "Chris" discovers the rule, then it becomes *Chris's* rule for addition of integers, not the teacher's.

What about Nancy's use of the calculator? Students who are using it as a quick way to merely get the answers to the problems are using it inappropriately. However, those using it to check their guesses for addition of large integers, or to find new patterns, are engaging the full power of the calculator to promote higher thinking. Nancy's arrangement of students' seats in pairs versus in rows of desks may be conducive to small group processing of ideas. *Principles and Standards* recommends that students work in small groups: "This approach is often very effective with students in the middle grades because they can try out their ideas in the relative privacy of a small group before opening themselves up to the entire class" (NCTM 2000, 272). In Nancy's groups, students worked individually applying her rules; therefore, there was little motivation for group members to share ideas. Furthermore, it was Nancy, not group members, who judged the correctness of answers and determined who would report answers on the board. She did not try to assess students' understanding, pose questions to provoke further thinking, or suggest to students that they enlist the help of others.

Was Nancy's approach bad? No, there might have been some educational gains for some students. Learners construct their own knowledge at all times and in all types of situations, but different instructional approaches may influence the quality and content accuracy of the construction. The fact that students faced each other in small groups rather than in rows looking at each others' backs surely promoted some worthwhile discussion among students, but the amount and quality of their interchange in terms of learning the mathematics involved would undoubtedly have been increased had Nancy designed the assignment to challenge students. Although the colored counters were not applied in the best way to enhance the students' ability to make connections between multiple representations, they still provided an alternative view for doing operations with integers, and they may have helped some students to better understand the mathematics. Nancy also had students present their answers, thus opening an opportunity for students to share their thinking and summarize ideas.

We surmise that Nancy's perception of teaching mathematics is one that relies on teacher control or is conceptually rule driven. She probably has had little experience using various tools, such as manipulatives, to guide exploratory activities. However, the fact that she has elements that are conducive to reform activities in her class indicates that she is trying to embrace different approaches to teaching. Her instruction and choice of activities are those of a teacher in transition to a reform-based teaching approach. A clearer vision of what the *Principles and Standards* recommends is a key to her success at moving forward with the transition.

Now let us consider a typical classroom of 30 students who are sitting in straight rows and busily working individually on a worksheet. Claudette, the teacher, stands at the front of the room or occasionally circulates about and looks over their shoulders. Is she teaching from a reform perspective? Maybe. It depends on what the worksheet requires and whether students have opportunities to learn in other ways described by the *Principles and Standards* on other days. Suppose Claudette's goal is for students to apply the heuristic "think of a simpler problem" to nonroutine problems. Below are the examples on the worksheet:

1. Find the last two digits of $11^{20} - 1$
2. Determine a rule for finding the following sum:
   $1^2 - 2^2 + 3^2 - 4^2 + 5^2 \ldots + 1999^2$
3. Be prepared to explain to the class your strategies for getting your answers.

The sheet is not of the "drill-and-kill" variety. It requires students to apply sound problem-solving heuristics to problems that are suitable for individual work. Furthermore, the third question promotes the sharing of students' ideas and discourse. If Claudette occasionally varies her teaching style, she may be teaching from a reform-based perspective.

The two examples above show that labeling an activity or class as reform-based or not requires close scrutiny of the work students do, how they do it, and whether a single teaching method is expected to be used exclusively.

EXEMPLARY PRACTICES

There are exemplary practices that clearly demonstrate the best practices for teaching and learning for understanding. For example, the teachers who are profiled in this book:

- Engage students in challenging, mathematically appropriate tasks that make sense to students.
- Create a classroom atmosphere conducive to discourse that encourages students' alternative conjectures, approaches, and explanations.
- Use appropriate tools, cooperative group work, and individual instruction to accommodate students with different learning styles.
- Use alternative assessment methods to assess students and guide their instruction.
- Collaborate with colleagues and pursue other professional development activities to support or improve their practice.

Do any of the teachers lecture at times? Sure. Many of us learned from lectures. (Of course, how well we understood what we learned is subject to debate.) Past NCTM President Gail Burrill (personal communication, April 1998) elaborates on perceptions that teachers should avoid when they attempt to implement reform:

> We must avoid misinterpretations such as: everything must be done in cooperative groups; decreased emphasis means none at all; every answer to every problem has to be explained in writing; the teacher is only a guide; every problem has to involve the real world; computational algorithms are not allowed; students should never practice; and manipulatives are the basis for all learning.
>
> The challenge is to make choices about content and teaching based on what we can do to enable students to learn.

In his article in the March 1997 issue of *The Mathematics Teacher* about applying common sense when implementing the *Principles and Standards*, classroom teacher Mark Saul echoes similar views:

> So what constitutes "real world?" As a classroom teacher, I have an operational definition: If it holds my students' attention, it is in their real world. If it does not, it is not. My job is to bring more mathematical thinking into their "real world." . . . What about technology? Should we not use calculators "at all times"? Well, no. We should be free to choose when to use them and when not to (183).

As mathematics educators, we know very well to be wary of universal statements such as "For all x, y is true." Both Burrill and Saul recommend that we be mindful that its negation, "There is an x for which y is false," is often true when x represents students in our classes and y represents a statement about the effectiveness of a specific activity or method. In essence, they are suggesting that we think as self-directed learners in the activities and strategies that we select to reach our students. If we do not heed their caution, I fear that education will continue to be entangled in radical movements that stress one philosophical stance over another.

Keeping our focus on all students' learning illuminates the fact that our students are too diverse to be neatly served by instructional methods labeled "Use me all the time!" The key reflective question that should guide whatever approach we take is, "How can we best facilitate students' understanding of the mathematical content in a meaningful way that contributes to their success in the twenty-first century?" Our best practices should align with the answers to that question.